1. 



Pictures NOT
accurately drawn
A 20 Euro note is a rectangle 133 mm long and 72 mm wide.
A 500 Euro Note is a rectangle 160 mm long and 82 mm wide.
Show that the two rectangles are not mathematically similar.
2.


The diagram shows a circle, centre $O$.
$A C$ is a diameter.
Angle $B A C=35^{\circ}$.
$D$ is the point on $A C$ such that angle $B D A$ is a right angle.
(a) Work out the size of angle $B C A$.

Give reasons for your answer.
$\qquad$ .${ }^{\circ}$
(b) Calculate the size of angle $D B C$.
$\qquad$ .
(c) Calculate the size of angle $B O A$.
3. $P Q R S$ is a quadrilateral.

$P Q$ is parallel to $S R$.
$S P$ is parallel to $R Q$.
(a) Prove that triangle $P Q S$ is congruent to triangle $R S Q$.
(b) In quadrilateral $P Q R S$, angle $S P Q$ is obtuse.

Explain why $P Q R S$ cannot be a cyclic quadrilateral.
4.


Diagram NOT<br>accurately drawn

Two cylinders, $\mathbf{P}$ and $\mathbf{Q}$, are mathematically similar.
The total surface area of cylinder $\mathbf{P}$ is $90 \pi \mathrm{~cm}^{2}$.
The total surface area of cylinder $\mathbf{Q}$ is $810 \pi \mathrm{~cm}^{2}$.
The length of cylinder $\mathbf{P}$ is 4 cm .
(a) Work out the length of cylinder $\mathbf{Q}$.
$\qquad$

The volume of cylinder $\mathbf{P}$ is $100 \pi \mathrm{~cm}^{3}$.
(b) Work out the volume of cylinder $\mathbf{Q}$.

Give your answer as a multiple of $\pi$.
$\mathrm{cm}^{3}$
5.


Diagram NOT
accurately drawn

The radius of the base of a cone is $x \mathrm{~cm}$ and its height is $h \mathrm{~cm}$.
The radius of a sphere is $2 x \mathrm{~cm}$.
The volume of the cone and the volume of the sphere are equal.
Express $h$ in terms of $x$.
Give your answer in its simplest form.
6.

$A B C$ is an equilateral triangle.
$A D$ is the perpendicular bisector of $B C$.
$B X$ is the angle bisector of angle $A B C$.
(a) Show that triangle $B X D$ is similar to triangle $A C D$.

In triangle $A C D$,
$A C=2 \mathrm{~cm}$,
$A D=\sqrt{3} \mathrm{~cm}$.
(b) Show that $X D=\frac{1}{\sqrt{3}} \mathrm{~cm}$.
7.

$A B$ is parallel to $D E$.
$A C E$ and $B C D$ are straight lines.
$A B=6 \mathrm{~cm}$,
$A C=8 \mathrm{~cm}$,
$C D=13.5 \mathrm{~cm}$,
$D E=9 \mathrm{~cm}$.
(i) Work out the length of $C E$.
cm
(ii) Work out the length of $B C$.
8.

Diagrams NOT accurately drawn


The two triangles $A B C$ and $P Q R$ are mathematically similar.

Angle $A=$ angle $P$.
Angle $B=$ angle $Q$.
$A B=8 \mathrm{~cm}$.
$A C=26 \mathrm{~cm}$.
$P Q=12 \mathrm{~cm}$.
$Q R=45 \mathrm{~cm}$.
(a) Work out the length of $P R$.
(b) Work out the length of $B C$.
9.

Diagram NOT accurately drawn

$A B C D$ is a square.
$B E C$ and $D C F$ are equilateral triangles.
(a) Prove that triangle $E C D$ is congruent to triangle $B C F$.
$G$ is the point such that $B E G F$ is a parallelogram.
(b) Prove that $E D=E G$
10.


Diagram NOT accurately drawn
Triangle $P Q R$ is isosceles with $P Q=P R$.
$X$ is a point on $P Q$.
$Y$ is a point on $P R$.
$P X=P Y$.
Prove that triangle $P Q Y$ is congruent to triangle $P R X$.
11.


Diagram NOT accurately drawn
In the diagram $Q$ and $R$ are points on the circumference of a circle.
$T Q$ and $T R$ are tangents to the circle.
$P Q=P R$.
Angle $R Q T=$ angle $Q R T=70^{\circ}$.
Angle $R P Q=120^{\circ}$.

Explain why $P$ is not the centre of the circle.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
12.

Diagram NOT
accurately drawn

$P$ and $Q$ are two points on a circle centre $O$.
The tangents to the circle at $P$ and $Q$ intersect at the point $T$.
(a) Write down the size of angle $O Q T$.
$\qquad$ .${ }^{\circ}$
(b) Calculate the size of the obtuse angle $P O Q$.
$\qquad$ .${ }^{\circ}$
(c) Give reasons why angle $P Q T$ is $70^{\circ}$
$\qquad$
$\qquad$

> 1. 1.84 .. $\neq 1.95$..
> $1.20 . . \neq 1.13$..
> $\frac{133}{72}=1.8472, \frac{160}{82}=1.9512$
> OR
> $\frac{72}{133}=0.54135, \frac{82}{160}=0.5125$
> OR
> $\frac{160}{133}=1.203 \ldots, \frac{82}{72}=1.1388 \ldots$.
> OR
> $\frac{133}{160}=0.83125 \ldots, \frac{72}{82}=0.878$
> M1 for $\frac{133}{72}$ ( $=1.8472 \ldots$ ) oe Accept 1.8, 1.85
> M1 for $\frac{160}{82}$ ( $=1.9512 \ldots$ ) oe consistent pairing
> Accept 2.0, 1.9
> OR M1 for $\frac{160}{133}$ (=1.203 ...) oe
> M1 for $\frac{82}{72}$ (=1.1388) oe
> Al for enough decimal places to show that the ratios are not equal; since the scale factors are different the shapes cannot be similar.
> NB Do Not need conclusion
2. (a) $55^{\circ}$
$90-35=55^{\circ}$
Angle in a semi-circle $=90^{\circ}$
B1 for $55^{\circ}$
B1 for (angle in) a semi-circle $=90^{\circ}$
(b) $35^{\circ}$

90 - " 55 "
B1 for $35^{\circ} f t$
(c) $110^{\circ}$
$180-2 \times 35=110^{\circ}$
M1 for complete method or for twice "(a)" Al cao
Candidates may choose to use Isosceles triangles or Angle subtended at centre is twice angle subtended at circumference
3. (a) Angle $\mathrm{PQS}=\mathrm{QSR}$

Angle RQS = PSQ
SQ is common
Triangles are congruent ASA
B1 for 1 condition + reason
B1 for $2^{\text {nd }}$ condition + reason
Bl for $3^{\text {rd }}$ condition + reason + statement of congruency
(b) Opposite angles of parallelograms are equal.

2 obtuse angle added are $>180^{\circ}$ therefore they cannot add up to 180 therefore the shape cannot be cyclic

B1 for states $P$ and $R$ are both obtuse
B1 sum greater than 180 for angles $P$ and $R$ or less than 180 for angles $Q$ and $S$
4. (a) 12

$$
\begin{aligned}
& \frac{810 \pi}{90 \pi} \text { or } 9 \\
& \sqrt{9} \text { or } 3
\end{aligned}
$$

M1 for $\frac{810 \pi}{90 \pi}$ or 9 or $\frac{1}{9}$ or $1: 9$ oe
M1 for $\sqrt{\frac{810 \pi}{90 \pi}}$ or $\sqrt{9}$ or 3 or $\frac{1}{3}$ or $\sqrt{9}: \sqrt{1}$ oe
Al cao
SC:B1 for answer of 36

$$
\begin{aligned}
& \text { (b) } 2700 \pi \\
& 3^{3} \text { or } 27 \text { or } 2700 \\
& \text { M1 for " } 3 \text { ",3 or } 27 \text { or }(\sqrt{9})^{3}:(\sqrt{81})^{3} \text { oe or } 9^{3} \text { or } 2700- \\
& \text { A1 cao }
\end{aligned}
$$2

5. $32 x$

$$
\begin{aligned}
& \frac{1}{3} \pi x^{2} h=\frac{4}{3} \pi(2 x)^{3} \\
& x^{2} h=4 \times 8 x^{3}
\end{aligned}
$$

M1 for substitution in correct formulae
M1 (dep.) for correct unsimplified expression eg
$h=\frac{\frac{4}{3} \pi(2 x)^{3}}{\frac{1}{3} \pi x^{2}}$ oe or $h=8 x$ oe
1 for $32 x$ cao
6. (a) Angle $B D X=$ angle $A D C=90^{\circ}$

Angle $B X D=$ angle $A C D=60^{\circ}$
Hence similar
B2 for 2 of (Angle $B D X=$ angle $A D C$, Angle $B X D=$ angle $A C D$, angle $D A C=$ angle $D B X$ )
B1 for 1 of the above
(b) $\frac{X D}{D C}=\frac{B D}{A D}, D C=B D=1$

M1 $\frac{X D}{D C}=\frac{B D}{A D}$ or $\frac{X D}{B D}=\frac{D C}{A D}$ or a statement that $A C D$ is an
enlargement of $B D X$, scale factor $\sqrt{3}$
Al $\frac{X D}{1}=\frac{1}{\sqrt{3}}$
A1 $X D=\frac{1}{\sqrt{3}}$
7. $\frac{C E}{8}=\frac{9}{6}$ or $\frac{C E}{9}=\frac{8}{6} \Rightarrow C E=\frac{8 \times 9}{6}$ $\frac{B C}{13.5}=\frac{6}{9}$ or $\frac{B C}{6}=\frac{13.5}{9} \Rightarrow B C=\frac{13.5 \times 6}{9}$
(i) 12
(ii) 9

M1 for scale factor $\frac{9}{6}\left(\right.$ or $\left.\frac{6}{9}\right)$ or $\frac{8}{6}\left(\operatorname{or} \frac{6}{8}\right)$ or $\frac{13.5}{9}\left(\right.$ or $\left.\frac{9}{13.5}\right)$ oe
Al cao for 12
Al cao for 9
8. (a) $\mathrm{SF}=1.5$ 39 cm
$M 1 S F=\frac{12}{8}, \frac{8}{12}, 1.5,0.6 \ldots$ oe
Al cao
(b) $45 \times \frac{8}{12}$

30 cm
M1 $45 \times \frac{8}{12}, 45 \div \frac{12}{8}$ oe
Al cao
9. (a) $B C=C E$ equal sides
$C F=C D$ equal sides
$B C F=D C E=150^{\circ}$
$B F C$ is congruent to $E C D$ (SAS)
Bl for either $B C=C D$ or $B C=C E$
$C F=C E$ or $C F=C D$
Bl for $B C F=D C E=150^{\circ}$ or correct reason
B1 for proof of congruence
(b) $\operatorname{So} B F=E D$ (congruent triangles)
$\mathrm{BF}=E G(\mathrm{opp}$ sides of parallelogram)
$B 1 B F=E G$ or $B F=E D$
B1 fully correct proof
10. $P Q=P R$ given
$P Y=P X$ given
Angle $P$ common
SAS

M1 for $P Q=P R$ with reason or $P Y=P X$ with reason
M1 for $P$ identified as a common angle
Al for completion of proof and SAS
11. $1 / 2(180-120)=30^{\circ}$

Angle $P Q R=30^{\circ}$ so $P Q$ is not a radius
M1 for angle between tangent and radius is $90^{\circ}$ or sight of right angle marked on diagram
A1 for angle $P Q R=30^{\circ}$ not $20^{\circ}$ or angle $P Q T=100^{\circ}$ not $90^{\circ}$ or $Q T R=40^{\circ}$ not $60^{\circ}$
12. (a) 90

B1 cao
(b) $140 \quad$ M1 for sight of $20^{\circ}$
or $360-90-90-40$
Al for $140^{\circ}$
SC: Award B1 for an answer or 220
(c)

B1 for Angle between tangent and radius $=\underline{90^{\circ}}$ or Tangents from a point to a circle are equal

B1 for Isosceles triangle $P O Q$ so angle $O Q P=20^{\circ}$ or Angles in a triangle add up to $180^{\circ}$

## 1. Mathematics $\mathbf{A}$

## Paper 4

Most answers showed no understanding of mathematical similarity, most making some comparison of areas or perimeters, or subtracted the lengths of the sides. Of those who did attempt a division, most gained the full marks, since they were then able to justify their results in the context of the question.

## Paper 6

Candidates did well on this unusual question. There were many successful approaches involving the calculation of appropriate scale factors and showing that they were not equal. Almost as popular was to calculate a scale factor from say $160 / 133$ and then multiplying 72 by this and showing the answer was not equal to 82 .

## Mathematics B Paper 17

The great majority of candidates did not understand the concept of similarity. Many simply worked out the areas of the two notes, offering no explanation. Some tried to use the monetary value of the notes to disprove similarity. A few did calculate ratios and explain their findings well.

## 2. Paper 3

In part (a) many candidates worked out the size of angle $B C A$ as $55^{\circ}$ but very few gave a valid reason. Part (b) was often done well, even after an incorrect part (a). Part (c) was answered less well with many candidates failing to realise that they were dealing with an isosceles triangle. A significant number of candidates did not appear to be familiar with angle notation and did not appreciate which angle they were expected to find in each part of this question.

## Paper 5

There was an improvement in the performance of candidates in this circle geometry question. $80 \%$ of candidates were able to calculate the missing angle in part (a) and about half of these candidates were able to correctly state a reason for their answer. Part (b) was also well answered with again $80 \%$ of candidates able to work out the correct answer. Part (c) was not so well answered as candidates had to make two steps in their solution. Only $57 \%$ of candidates were successful in this part.
3. Only $7 \%$ of candidates were able to give a full solution to proving that the two triangles were congruent in part (a); they often took for granted what they were trying to prove. In part (b) candidates did not give sufficient reasons as to why two obtuse angles had a swn of greater than $180^{\circ}$ though $20 \%$ of candidates gave a complete solution and a further $10 \%$ a partial solution,

## 4. Specification $\mathbf{A}$

Most candidates attempted this question but only the best were able to achieve full marks. By far the most common answer to part (a) was 36 , and to part (b) was $900 \pi$.
In part (b), some candidates were able to score a mark for cubing the scale factor they derived in part (a). A small number of candidates calculated the radius of $P$ to deduce the radius, and hence the volume, of $Q$. A significant proportion of those candidates getting as far as the volume $2700 \pi$ did not understand the demand of the question and omitted to include the $\pi$ in their final answer.

## Specification B

The most commonly seen answer to this question was 36 in (a) and $900 \pi$ in (b). These were both incorrect solutions and occurred when candidates mistakenly used the area scale factor as the length and then volume scale factor. A small minority of candidates were able to often fully correct solutions although the omission of $\pi$ from the answer to (b) resulted in some candidates failing to gain the final accuracy mark. There was evidence of some poor arithmetic in this question with 810 divided by 90 being evaluated as 90 .

## 5. Specification $\mathbf{A}$

Although there were few completely correct answers to this question, many candidates scored at least one mark for correctly using the equations for the cone and sphere.
A typical candidate substituted $x$ into the equation for the cone, $2 x$ into the equation for the sphere, forgot to cube the 2 and, after rearrangement, arrived at an answer $8 x$.
There were a surprising number of candidates who, having started correctly with $\frac{1}{3} \pi x^{2} h=$ $\frac{4}{3} \pi(2 x)^{3}$, than incorrectly subtract the fractions to get $\pi x^{2} h=1 \pi(2 x)^{3}$.

## Specification B

Approximately two thirds of candidates were able to substitute the given radii into the correct formulae. The formula for the surface area of a sphere was sometimes used instead of the formula for the volume. The most common error on substituting the radii was to give $\frac{4}{3} p 2 x^{3}$ instead of the correct $\frac{4}{3} p(2 x)^{3}$. Candidate who made this error lost one mark out of the available three provided they managed to make $h$ the subject of their formula using correct algebraic processes. The level of algebra was generally very poor with very few candidates being able to carry out what should have been a straightforward division. The most common error here was to subtract $\frac{1}{3}$ from either side of the equation rather than multiply by 3 .
6. There was a great deal of confusion between conditions for similarity and conditions for congruence., with such 'explanations' as 'Angle, angle, side' or 'right angle, hypotenuse and angle' being quoted. It was a pleasure to see good attempts at part (b), mainly using the similarity established in part (a) but with a few using the fact that $\tan 30^{\circ}=\frac{1}{\sqrt{3}}$ which can be obtained from triangle $A D C$.

## 7. Intermediate Tier

This question was answered poorly. Candidates either have very little understanding of similar triangles or fail to recognize them when they appear. Many gave the two lengths as 13.5 cm and 8 cm (assuming the triangles to be isosceles) and some attempted to use Pythagoras' theorem. About $10 \%$ of candidates got at least one length correct but few wrote down a scale factor.

## Higher Tier

Many candidates did well on this question. Most were able to match the triangles to derive the appropriate ratios. Generally candidates who could do (i) could also do (ii). There were a relatively small number of candidates who attempted to use Pythagoras' theorem or the cosine rule for the lengths.

## 8. Specification A

## Intermediate Tier

The common approach to this question was to assume that values were added on to give the enlarged triangle, rather than the adoption of an approach which involved factors. A few attempted to apply Pythagoras. Credit was available for finding factors alone, but there was little evidence to substantiate the award of these method marks. It was clear that most candidates failed to associate "similar" with scale factors.

## Higher Tier

Many candidates scored full marks on this question, and there were relatively few incorrect methods involving Pythagoras' theorem. Common mistakes occurred in simplifying the scale factor $\frac{12}{8}$ to $\frac{4}{3}$ or 1.4 ; and in part (b), errors in calculating $45 \div 1.5$. Some of the weaker candidates, not understanding the need for a scale factor, simply added 4 (derived from the difference of $P Q$ and $A B$ ) to $A C$ to obtain $R P=30$.

## Specification B

## Intermediate tier

It was disappointing to see so many candidates merely adding or subtracting 4 cm from the given lengths to give answers of $P R=30(26+4)$ and $B C=41(45-4)$. A small number used RQ in (a) instead of (b), and AC likewise; they then contrived to get the desired results by wrongly manipulating the measurements given on the diagram. This gained no marks. Few candidates quoted any scale factor of enlargement although some did use the "once and a half again" method, usually to good effect. Very few quoted the equivalence of the ratios of corresponding sides. Quite a few tried to use Pythagoras ignoring that the question did not have right angled triangles.
9. Setting out these proofs was not done well. Many used very wordy explanations rather than concise mathematical reasons and failed to identify the key stages required. About half the candidates were able to score at least one mark in part (a), usually for identifying a pair of equal sides in the triangles. Candidates often failed to give a reason for the equal angles (this was merely stated as fact), and some failed to identify the appropriate congruence. Generally part (b) was done a little better than part (a), but many candidates identified EDG as an equilateral triangle, or stated that EDF and EGF were identical isosceles triangles, without supporting evidence or explanation.
10. Few candidates understood the nature of proof. This question was very poorly done. it is important that candidates do recognise that they must qualify any statements they make. It was not sufficient to write $P R=P Q$ without including that this was because triangle $P Q R$ was isosceles or that the information was given.
11. This question was very poorly answered with $80 \%$ of candidates scoring no marks. It was a pleasant surprise when candidates knew what they were talking about and managed to convey their ideas across...if not always eloquently. The tangent/ radius connection was often stated in about $10 \%$ of cases and most went on to discuss the $20^{\circ} / 30^{\circ}$ situation, though some did get confused expecting the $70^{\circ}$ to be $90^{\circ}$. Only $10 \%$ of candidates were able to produce a fully reasoned argument.
12. It was disappointing to find that only just over half the candidates realised that angle $O Q T$ was $90^{\circ}$. This, in turn, went on to affect their calculations in part (b) as they could then have gone on to use the sum of the angles of the quadrilateral in an attempt to work out the size of angle $P O Q$ or recognise that $O P Q$ was an isosceles triangle with the acute angles being $20^{\circ}$ each. The mean mark for this part of the question was just under 1. Candidates often struggle to provide mathematical reasons for the size of an angle and this year was no exception with 0.37 being the mean mark for this part. Those who did score one of the marks tended to score it for mention of an isosceles triangle or for stating that the sum of the angles of a triangle is $180^{\circ}$. The few that did mention tangent theorems tended to talk about the angle between the tangent and the circle rather than the tangent and the radius.

